

# tzplot: Graphs in Economics

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# 1 Axes

```
\tzaxes(0,0)(10,9){$x$}{$y$}
```

```
\tzaxisx{0}{10}{$x$}
```

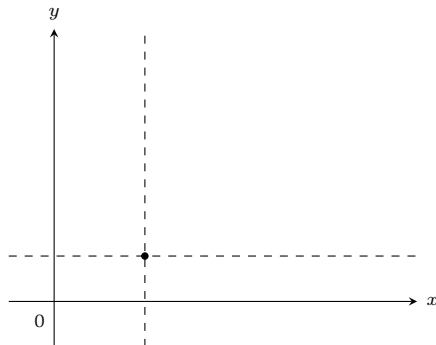
```
\tzaxisy{0}{9}{$y$}
```

```
% shift
```

```
\tzaxes<2,1>(0,0)(10,9){$x$}{$y$}
```

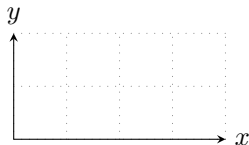
```
\tzaxisx<2>{0}{10}{$x$}
```

```
\tzaxisy<1>{0}{9}{$y$}
```

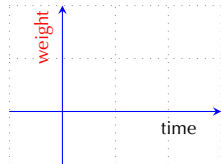


## 1.1 \tzaxes

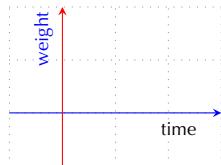
```
\begin{tikzpicture}[scale=.7]
\tzhelplines(4,2)
\tzaxes(4,2){$x$}{$y$}
\end{tikzpicture}
```



```
% \tzplot: intersects at (0,0)
\begin{tikzpicture}[scale=.7,font=\scriptsize]
\tzhelplines(-1,-1)(3,2)
\tzaxes[draw=blue](-1,-1)(3,2)
    {time}[b,pos=.8]{weight}[a,pos=.8,sloped,red]
\end{tikzpicture}
```

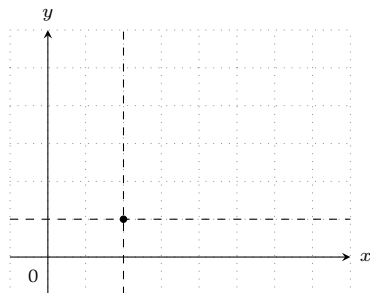


```
% \tzaxisx, \tzaxisy
\begin{tikzpicture}[scale=.7,font=\scriptsize]
\tzhelplines(-1,-1)(3,2)
\tzaxisx[draw=blue]{-1}{3}{time}[b,pos=.8]
\tzaxisy[draw=red]{-1}{2}{weight}[a,pos=.8,sloped,blue]
\end{tikzpicture}
```

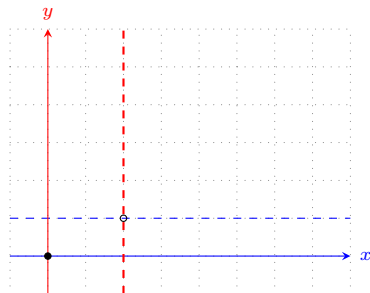


## 1.2 Shift

```
% \tzaxes: intersects at <2,1>
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-1,-1)(8,6)
\tzshoworigin
\tzaxes(-1,-1)(8,6){$x$}{$y$}
\tzaxes[-,dashed]<2,1>(-1,-1)(8,6)%{$x$}{$y$}
\tzdot*(2,1)
\end{tikzpicture}
```

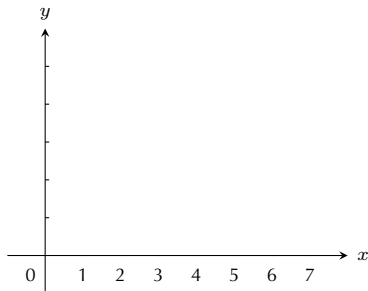


```
% shift: \tzaxisx, \tzaxisy
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-1,-1)(8,6)
\tzaxisx[blue]{-1}{8}{$x$}
\tzaxisx[-,dashed,blue]<1>{-1}{8}%{$x$}
\tzaxisy[red]{-1}{6}{$y$}
\tzaxisy[-,dashed,red,thick]<2>{-1}{6}%{$y$}
\tzshoworigin*
\tzdot(2,1)
\end{tikzpicture}
```



## 2 Ticks

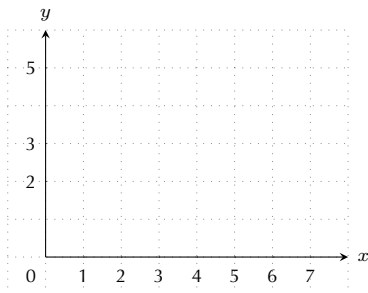
```
% tick labels
\zticks{1,2,...,10}{1,2,...,9}
\zticksx{1,2,...,10}
\zticksy{1,2,...,9}
```



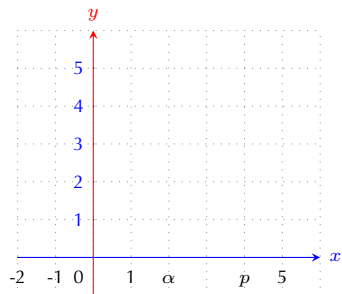
```
% ticks* (length-from:to) : no labels
\zticks*(0:2mm){1,2,...,10}(-1mm:2mm){1,2,...,9}
\zticksx*(0:2mm){1,2,...,10}
\zticksy*(-1mm:2mm){1,2,...,9}
```

## 2.1 \zticks: Tick labels

```
% \zticks
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-1,-1)(8,6)
\tzshoworigin
\tzaxes(8,6){ $x$ }{ $y$ }
\zticks{1,2,\dots,7}{2,3,5}
\end{tikzpicture}
```



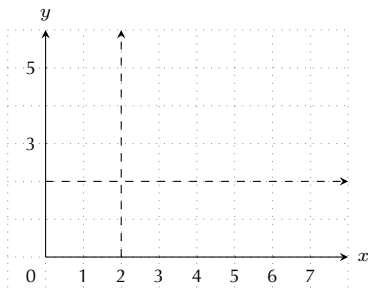
```
% \zticksx, \zticksy
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-2,-1)(6,6)
\tzshoworigin
\tzaxisx[blue]{-2}{6}{ $x$ }
\zticksx{-2,-1,1,2/{\alpha},4/{p},5}
\tzaxisy[red]{-1}{6}{ $y$ }
\zticksy[blue]{1,2,\dots,5}
\end{tikzpicture}
```



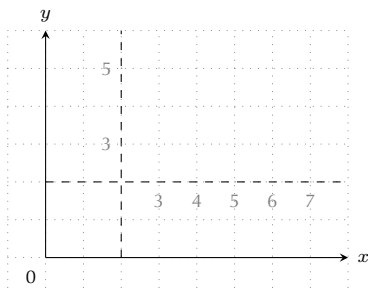


## 2.2 Shift

```
% \tzticks
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-1,-1)(8,6)
\tzshoworigin
\tzaxes(8,6){$x$}{$y$}
\tzticks{1,2,...,7}{3,5}
\tzaxes[dashed]<2,2>(8,6)%{$x$}{$y$}
\end{tikzpicture}
```

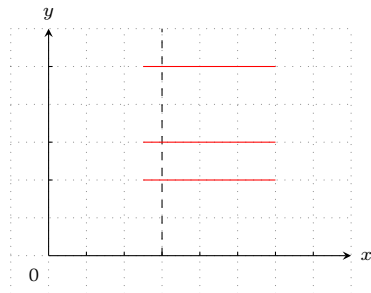


```
% \tzticks : shift
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-1,-1)(8,6)
\tzshoworigin
\tzaxes(8,6){$x$}{$y$}
%\tzticks{1,2,...,7}{3,5}
\tzaxes[-,dashed]<2,2>(8,6)%{$x$}{$y$}
\tzticks[gray]<2,2>{3,4,...,7}{3,5}
\end{tikzpicture}
```

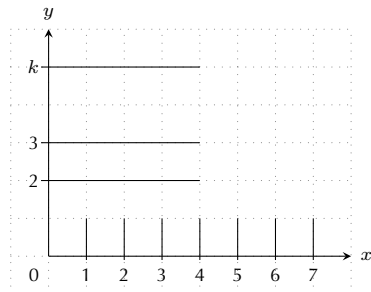


## 2.3 \zticks\*: Ticks

```
% \zticks* : no labels
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-1,-1)(8,6)
\tzshoworigin
\tzaxes(8,6){$x$}{$y$}
\zticks*{1,2,...,7}{2,3,5/$k$} % default: (Opt:3pt)
\tzaxisy[-,dashed]<3>{0}{6}
\zticksy*[red]<3>(-5mm:3cm){2,3,5/$k$} % change length
\end{tikzpicture}
```



```
% \zticks : ticks and labels
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-1,-1)(8,6)
\tzshoworigin
\tzaxes(8,6){$x$}{$y$}
\zticks(-1mm:1cm){1,2,...,7}
(-2mm:4cm){2,3,5/$k$}
\end{tikzpicture}
```



### 3 Functions

```
\tzfn{\Fx}[1:5]
```

```
\tzfn\Fx[1:5] % simple form
```

```
% inverse function
```

```
\tzfn'\Fx[1:5] % swap version
```

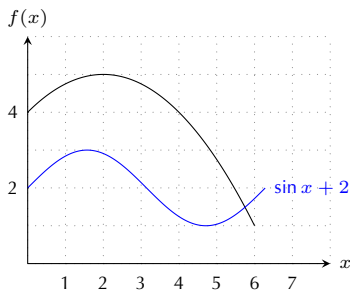
```
% function of \y
```

```
\tzfnofy\Fy[1:5]
```

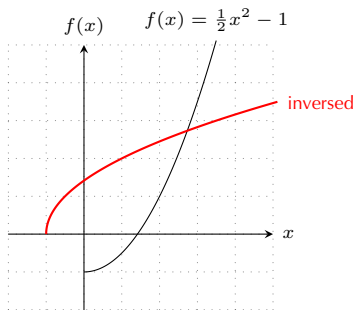
```
\tzfnofy'\Fy[1:5]
```

### 3.1 \tzfn and \tzfn'

```
% \tzfn
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6){$x$}{$f(x)$}
\tzfn{-.25*(\x-2)^2+5}[0:6]
\tzfn[blue]{sin(\x r)+2}[0:2*pi]
    {${\sin x+2}$}[r]
\tzticks{1,2,...,7}{2,4}
\end{tikzpicture}
```

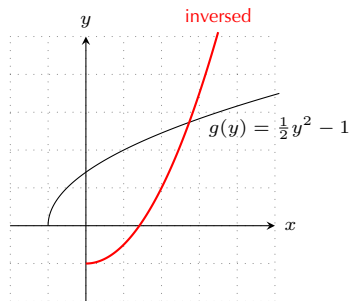


```
% \tzfn' : inverse function
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-2,-2)(5,5)
\tzaxes(-2,-2)(5,5){$x$}{$f(x)$}
\def\Fx{.5*(\x)^2-1}
\tzfn\Fx[0:3.5]{$f(x)=\frac{1}{2}x^2-1$}[a]
\tzfn'[red,thick]\Fx[0:3.5]{inversed}[r] %%
\end{tikzpicture}
```

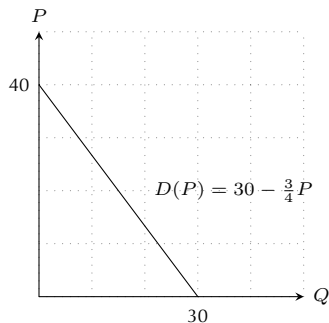


## 3.2 \tzfnofy(')

```
% \tzfnofy(')
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(-2,-2)(5,5)
\tzaxes(-2,-2)(5,5){$x$}{$y$}
\def\Fy{.5*(\y)^2-1} % function of \y
\tzfnofy\Fy[0:3.5]{$g(y)=\frac{1}{2}y^2-1$}[b=5pt]
\tzfnofy'[red,thick]\Fy[0:3.5]{inversed}[a] %%
\end{tikzpicture}
```

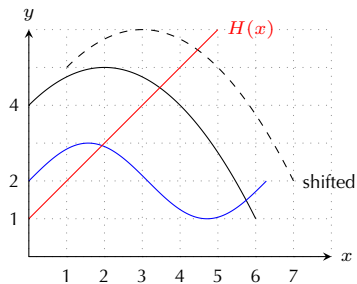


```
% \tzfnofy: demand function
\begin{tikzpicture}[scale=.07,font=\scriptsize]
\tzhelplines[step=10](50,50)
\tzaxes(50,50){$Q$}{$P$}
\def\Fy{30-3/4*\y} % function of \y
\tzfnofy\Fy[0:40]
\tznode(20,20){$D(P)=30-\frac{3}{4}P$}[r]
\tzticks{30}{40}
\end{tikzpicture}
```



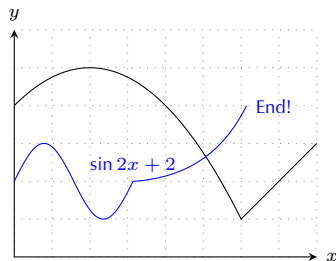
### 3.3 Shift

```
% \tzfn<shift> : shift
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6){$x$}{$y$}
\def\Fx{-.25*(\x-2)^2+5}
\def\Gx{sin(\x r)+2}
\def\Hx{1+\x}
\tzfn\Fx[0:6] %% simple
\tzfn[dashed]<1,1>\Fx[0:6]{shifted}[r] %% shift
\tzfn[blue]\Gx[0:2*pi]
\tzfn[red]\Hx[0:5]{$H(x)}[r]
\tzticks{1,2,...,7}{1,2,4}
\end{tikzpicture}
```

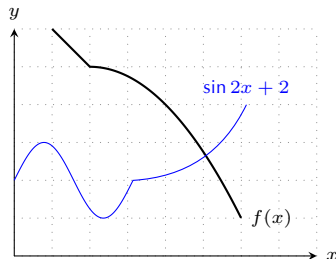


### 3.4 Extending paths

```
% \tzfn\Fx[0:6]<code.append>
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6){$x$}{$y$}
\def\Fx{- .25*(\x-2)^2+5}
\def\Gx{sin(2*\x r)+2}
\tzfn\Fx[0:6]<-- ++ (2,2)> %%
\tzfn[blue]\Gx[0:pi]{$\sin 2x+2$}[a]
<to[bend right]++(3,2) node [r] {End!}> %%
\end{tikzpicture}
```



```
% \tzfnAtBegin, \tzfnAtEnd
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6){$x$}{$y$}
\def\Fx{- .25*(\x-2)^2+5}
\def\Gx{sin(2*\x r)+2}
\tzfnAtBegin{(1,6)--} %%
\tzfn[thick]\Fx[2:6]{f(x)}[r]
\tzfnAtEnd{to[bend right]++(3,2)} %%
\tzfn[blue]\Gx[0:pi]{$\sin 2x+2$}[a]
\end{tikzpicture}
```



## 4 More functions

`\tzhfnat{<y>}[xmin:xmax] % horizontal`

`\tzhfn(<x,y>)[xmin:xmax]`

`\tzvfnat{<x>}[ymin:ymax] % vertical`

`\tzvfn(<x,y>)[ymin:ymax]`

`\tzLFn(coor1)(coor2){<slope>}[xmin:xmax]`

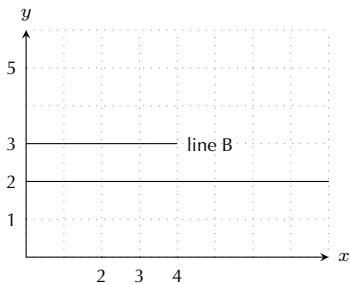
`\tzfnmax{\Fx,\Gx}[xmin:xmax]`

`\tzfnmin{\Fx,\Gx}[xmin:xmax]`

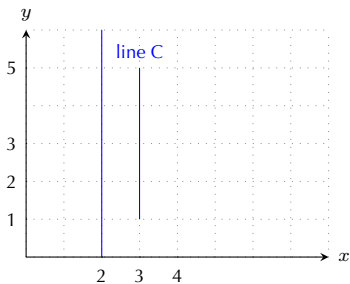


## 4.1 \tzhfn(at) and \tzvfn(at)

```
% \tzhfn(at) : horizontal
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines*(8,6)
\tzaxes(8,6){\x$}{\y$}
\tzhfnat{2} % at y=2
\tzhfn(8,3)[0:4]{line B}[r] % at y=3, ignores x
\tzticks{2,3,4}{1,2,3,5}
\end{tikzpicture}
```

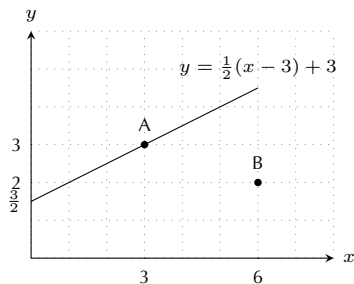


```
% \tvhfn(at) % vertical
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines*(8,6)
\tzaxes(8,6){\x$}{\y$}
\tzvfnat[blue]{2} % at x=2
\tzvfn[blue](3,8)[1:5]{line C}[a] % at x=3, ignores y
\tzticks{2,3,4}{1,2,3,5}
\end{tikzpicture}
```

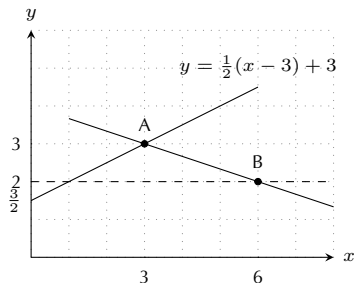


## 4.2 \tzLFn

```
% \tzLFn(coor){slope}
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines*(8,6)
\tzaxes(8,6){$x$}{$y$}
\tzcoors*(3,3)(A){A}(6,2)(B){B};
\tzLFn(A){.5}[0:6]{$y=\frac{1}{2}(x-3)+3$}[a]
\tzticks{3,6}{1.5/$\frac{3}{2}$,2,3}
\end{tikzpicture}
```

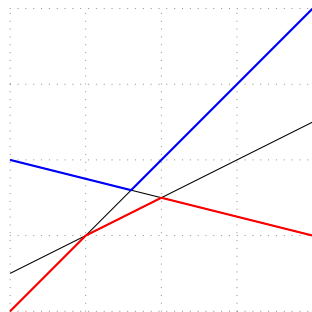


```
% \tzLFn(coor)(coor)
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines*(8,6)
\tzaxes(8,6){$x$}{$y$}
\tzcoors*(3,3)(A){A}(6,2)(B){B};
\tzLFn(A)(B)[1:8]
\tzLFn(A){.5}[0:6]{$y=\frac{1}{2}(x-3)+3$}[a]
\tzticks{3,6}{1.5/$\frac{3}{2}$,2,3}
\tzhfn[dashed](B) %% ignores (B.x)
\end{tikzpicture}
```

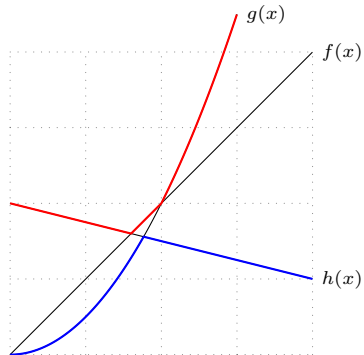


### 4.3 \tzfnmax and \tzfnmin

```
% \tzfnmin, \tzfnmax: envelope curves
\begin{tikzpicture}
\tzhelplines*(4,4)
\def\Fx{\x} \def\Gx{.5*\x+.5} \def\Hx{-.25*\x+2}
\tzfn\Fx[0:4]
\tzfn\Gx[0:4]
\tzfn\Hx[0:4]
\tzfnmin[red,thick]{\Fx,\Gx,\Hx}[0:4]
\tzfnmax[blue,thick]{\Fx,\Gx,\Hx}[0:4]
\end{tikzpicture}
```



```
% \tzfnmin, \tzfnmax: envelope curves
\begin{tikzpicture}[font=\scriptsize]
\tzhelplines*(4,4)
\def\Fx{\x} \def\Gx{.5*(\x)^2} \def\Hx{-.25*\x+2}
\tzfn\Fx[0:4]{$f(x)$}[r]
\tzfn\Gx[0:3]{$g(x)$}[r]
\tzfn\Hx[0:4]{$h(x)$}[r]
\tzfnmin[samples=501,blue,thick]{\Fx,\Gx,\Hx}[0:4]
\tzfnmax[samples=501,red,thick]{\Fx,\Gx,\Hx}[0:3]
\end{tikzpicture}
```



## 5 Intersections

```
\tzXpoint{pathA}{pathB}(<coord name>)  
\tzXpoint*{pathA}{pathB}(<coord name>)
```

```
% horizontal intersections
```

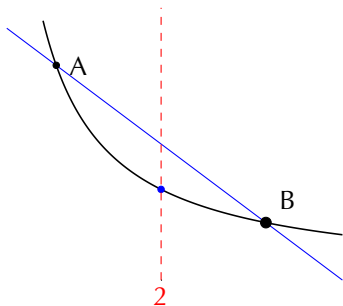
```
\tzhXpointat{path}{<y>}
```

```
\tzhXpoint{path}(<x,y>)
```

```
% vertical intersections
```

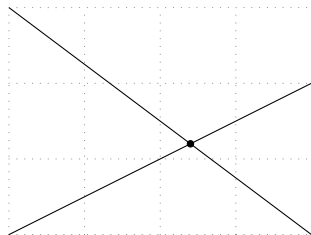
```
\tzvXpointat{path}{<x>}
```

```
\tzvXpoint{path}(<x,y>)
```

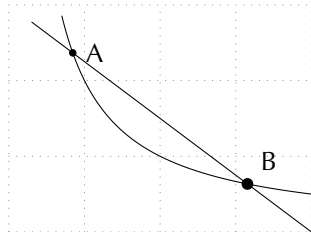


## 5.1 \tzXpoint of two paths

```
% \tzXpoint
% "<path name>"
\begin{tikzpicture}
\tzhelplines(4,3)
\tzline"line A"(0,0)(4,2) % [name path=line A]
\tzline"line B"(0,3)(4,0) % [name path=line B]
\tzXpoint{line A}{line B}(K)
\tzdot*(K)
\end{tikzpicture}
```

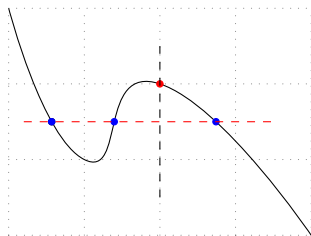


```
% \tzfn and "path name"
\begin{tikzpicture}
\tzhelplines(4,3)
\def\Fx{3-3/4*\x}
\def\Gx{2/\x}
\tzfn"Fx"\Fx[.3:4] % [name path=Fx]
\tzfn \Gx[.7:4] % [name path=Gx] automatically
\tzXpoint{Fx}{Gx}(K)
\tzdot*(K-1){A}[0]
\tzdot*(K-2){B}[45] (4pt)
\end{tikzpicture}
```

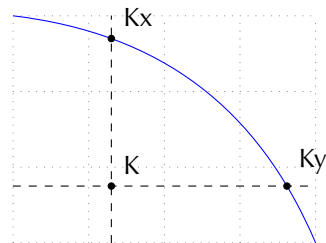


## 5.2 `\tzhXpoint(at)` and `\tzvXpoint(at)`

```
% \tzhXpointat, \tzvXpointat
\begin{tikzpicture}
\tzhelplines(4,3)
\tzplotcurve"curve"(0,3)(1,1)(2,2)(4,0);
\tzhXpointat*{curve}{1.5}(X)
\tzdots*[blue](X)(X-2)(X-3);
\tzhfn[red,dashed](X)[.2:3.5]
\tzvXpointat*[red]{curve}{2}(Y) % dot
\tzvfndashed(Y)[.5:2.5]
\end{tikzpicture}
```



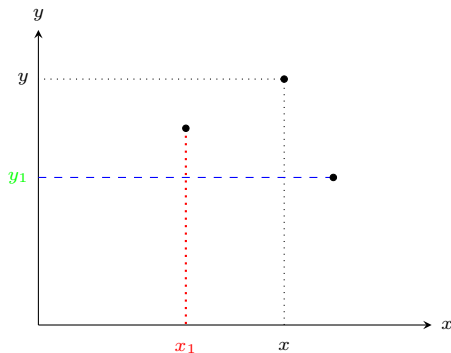
```
% \tzhXpoint, \tzvXpoint
\begin{tikzpicture}
\tzhelplines*(4,3)
\tzcoor*(30:1.5cm)(K){K}[45]
\tzto[blue,bend left]"curve"(0,3)(4,0)
\tzhXpoint*{curve}(K)(Ky){Ky}[45]
\tzvXpoint*{curve}(K)(Kx){Kx}[45]
\tzhfn[dashed](K)
\tzvfndashed(K)
\end{tikzpicture}
```



## 6 Projections

`\tzproj(<coord>)`

`\tzproj*(<coord>)`



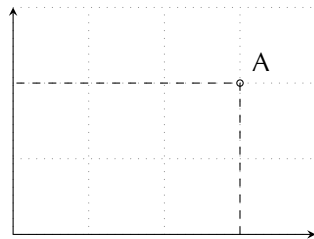
`\tzproj(<coord>){<x-tick label>}{<y-tick label>}`

`\tzprojx(<coord>){<x-tick label>}`

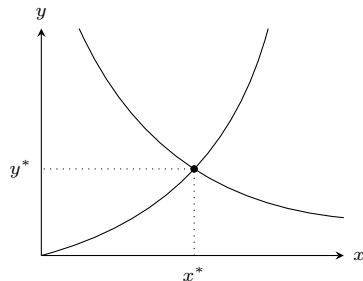
`\tzprojy(<coord>){<y-tick label>}`

## 6.1 \tzproj

```
% \tzproj
\begin{tikzpicture}
\tzhelplines(4,3)
\tzaxes(4,3)
\tzcoor*[fill=none](3,2)(A){A}[45]
\tzproj[dashed](A)
\end{tikzpicture}
```



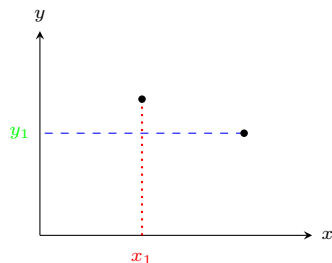
```
% \tzproj* : dot
\begin{tikzpicture}[font=\scriptsize]
\tzhelplines(4,3)
\tzaxes(4,3){$x$}{$y$}
\tzto[bend right]"dem"(.5,3)(4,.5)
\tzto[bend right]"sup"(0,0)(3,3)
\tzXpoint{dem}{sup}(E)
\tzproj*(E){$x^*$}{$y^*$}
\end{tikzpicture}
```



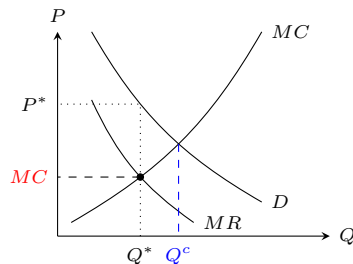


## 6.2 \tzprojx and \tzprojy

```
% \tzprojx*, \tzprojy*
\begin{tikzpicture}[scale=.9,font=\scriptsize]
\tzaxes(4,3){$x$}{$y$}
\tzprojx*[red,thick](1.5,2){$x_1$}
\tzprojy*[blue,dashed](3,1.5){$y_1$}[green]
\end{tikzpicture}
```



```
% \tzprojx(*), \tzprojy(*)
\begin{tikzpicture}[scale=.9,font=\scriptsize]
\tzaxes(4,3){$Q$}{$P$}
\tzto[bend right=15]"DD"(.5,3)(3,.5){$D$}[r]
\tzto[bend right=15]"MC"(.2,.2)(3,3){$MC$}[r]
\tzto[bend right=15]"MR"(.5,2)(2,.2){$MR$}[r]
\tzXpoint{MR}{MC}(E)
\tzvXpoint{DD}(E)(EP)
\tzproj(EP){$Q^*$}{$P^*$}
\tzprojy*[dashed](E){$MC$}[red]
\tzXpoint{DD}{MC}(EC)
\tzprojx[blue,dashed](EC){$Q^c$}
\end{tikzpicture}
```



## 7 Slopes

`\tzsecantat{path}{<x1>}{<x2>}`

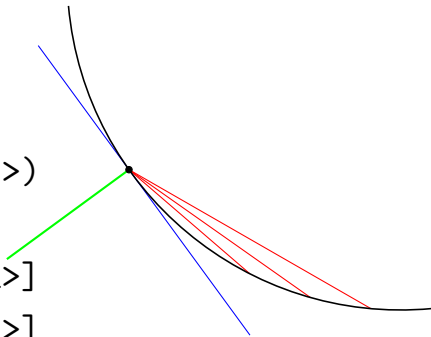
`\tzsecant{path}(<x1,y1>)(<x2,y2>)`

`\tztangentat{path}{<x>}[<domain>]`

`\tztangent{path}(<x,y>)[<domain>]`

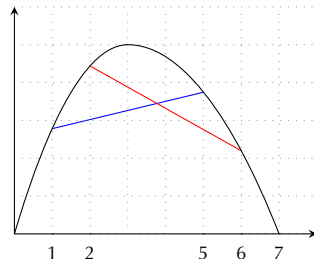
`\tzslopeat{path}{<x>}{<length>}[<rotate>]`

`\tzslope{path}(<x,y>){<length>}[<rotate>]`

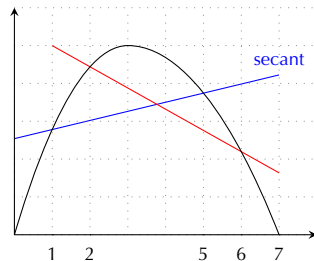


## 7.1 Secant lines: \tzsecant(at)

```
% \tzsecantat  
\begin{tikzpicture}[scale=.5,font=\scriptsize]  
\tzhelplines(8,6)  
\tzaxes(8,6)  
\tzparabola"curve"(0,0)(3,5)(7,0)  
\tzsecantat[blue]{curve}{1}{5}  
\tzsecantat[red]{curve}{2}{6}  
\zticksx{1,2,5,6,7}  
\end{tikzpicture}
```



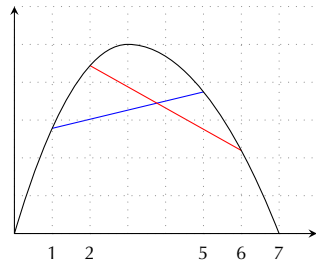
```
% \tzsecantat with [domain]  
\begin{tikzpicture}[scale=.5,font=\scriptsize]  
\tzhelplines(8,6)  
\tzaxes(8,6)  
\tzparabola"curve"(0,0)(3,5)(7,0)  
\tzsecantat[blue]{curve}{1}{5}[0:7]{secant}[a]  
\tzsecantat[red]{curve}{2}{6}[1:7]  
\zticksx{1,2,5,6,7}  
\end{tikzpicture}
```



```

% \tzsecant
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6)
\tzparabola"curve"(0,0)(3,5)(7,0)
\tzsecant[blue]{curve}(1,0)(5,0)
\tzsecant[red]{curve}(2,0)(6,0)
\tzticksx{1,2,5,6,7}
\end{tikzpicture}

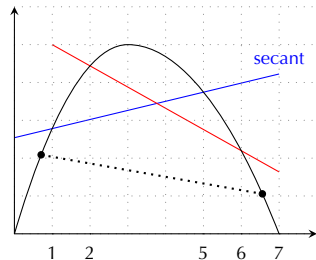
```



```

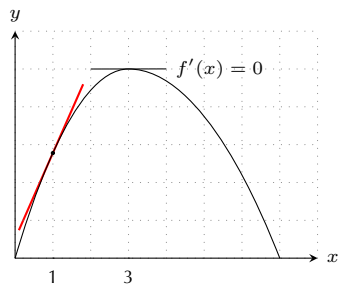
% \tzsecant with [domain]
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6)
\tzparabola"curve"(0,0)(3,5)(7,0)
\tzsecant[blue]{curve}(1,0)(5,0)[0:7]{secant}[a]
\tzsecant[red]{curve}(2,0)(6,0)[1:7]
\tzticksx{1,2,5,6,7}
\tzcoors({sqrt(.5)},0)(A)({sqrt(43)},0)(B);
\tzvXpoint*{curve}(A)
\tzvXpoint*{curve}(B)
\tzsecant[dotted,thick]{curve}(A)(B)
\end{tikzpicture}

```

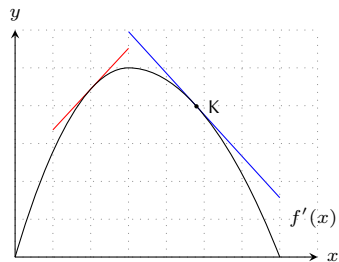


## 7.2 Tangent lines: \tztangent(at)

```
% \tztangentat
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6){$x$}{$y$}
\tzparabola"curve"(0,0)(3,5)(7,0)
\tzvXpointat*{curve}{1}(K)(1.2pt)
\tztangentat[red,thick]{curve}{1} [.1:1.8]
\tztangentat{curve}{3}[2:4]{$f'(x)=0$}[r]
\tzticksx{1,3}
\end{tikzpicture}
```



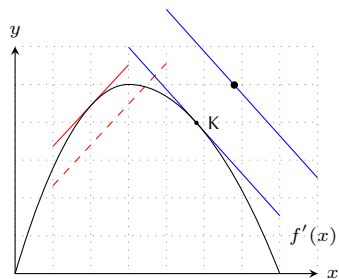
```
% \tztangent
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelplines(8,6)
\tzaxes(8,6){$x$}{$y$}
\tzparabola"curve"(0,0)(3,5)(7,0)
\tztangent[red]{curve}(2,0)[1:3]
\tzvXpointat*{curve}{sqrt(23)}(K){K}[0](1.2pt)
\tztangent[blue]{curve}(K)[3:7]{$f'(x)$}[br]
\end{tikzpicture}
```



```

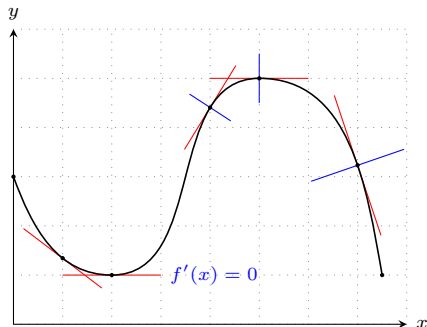
% \tztangent : shift
\begin{tikzpicture}[scale=.5,font=\scriptsize]
\tzhelpelines(8,6)
\tzaxes(8,6){$x$}{$y$}
\tzparabola"curve"(0,0)(3,5)(7,0)
\tztangent[red]{curve}(2,0)[1:3]
\tztangent[red,dashed]<.5,-.5>{curve}(2,0)[.5:3.5] %%
\tzvXpointat*{curve}{sqrt(23)}(K){K}[0](1.2pt)
\tztangent[blue]{curve}(K)[3:7]{$f'(x)$}[br]
\tztangent[blue]<1,1>{curve}(K)[3:7]{$f'(x)$}[br] %%
\tzdot*<1,1>(K) %% checking
\end{tikzpicture}

```



### 7.3 \tzslope(at)

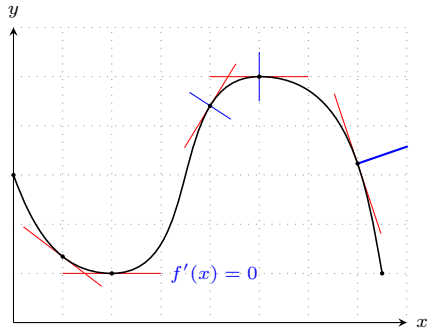
```
% \tzslopeat
\begin{tikzpicture}[scale=.65,font=\scriptsize]
\settzdotsize{1.2pt}
\tzhelplines(8,6)
\tzaxes(8,6){$x$}{$y$}
\tzcoors*(0,3)(A)(2,1)(B)(5,5)(C)(7.5,1)(D);
\tztos[semithick]"curve"
(A)[out=-70,in=180] (B)[out=0,in=180]
(C)[out=0,in=100] (D);
\tzvXpointat*{curve}{1}
\tzvXpointat*{curve}{4}
\tzvXpointat*{curve}{7}
\tzslopeat[red]{curve}{1}{2cm}
\tzslopeat[red]{curve}{2}{2cm}{f'(x)=0}[r,blue]
\tzslopeat[red]{curve}{4}{2cm}
\tzslopeat[red]{curve}{5}{2cm}
\tzslopeat[red]{curve}{7}{3cm}
\tzslopeat[blue]{curve}{4}{1cm}[90] % normal
\tzslopeat[blue]{curve}{5}{1cm}[90] % normal
\tzslopeat[blue]{curve}{7}{2cm}[90] % normal
\end{tikzpicture}
```



```

% \tzslope
\begin{tikzpicture}[scale=.65,font=\scriptsize]
\settzdotsize{1.2pt}
\tzhelplines(8,6)
\tzaxes(8,6){\x$}{\y$}
\tzcoors*(0,3)(A)(2,1)(B)(5,5)(C)(7.5,1)(D);
\tztos[semithick]"curve"
(A)[out=-70,in=180] (B)[out=0,in=180]
(C)[out=0,in=100] (D);
\tzvXpointat*{curve}{1}
\tzvXpointat*{curve}{4}
\tzvXpointat*{curve}{7}
\tzslope[red]{curve}(1,0){2cm}
\tzslope[red]{curve}(2,0){2cm}{ $f'(x)=0$ }[r,blue]
\tzslope[red]{curve}(4,0){2cm}
\tzslope[red]{curve}(5,0){2cm}
\tzslope[red]{curve}(7,0){3cm}
\tzslope[blue]{curve}(4,0){1cm}[90] % normal
\tzslope[blue]{curve}(5,0){1cm}[90] % normal
\tzslope[blue,thick,tzextend={0pt}{7mm}]
{curve}(7,0){.1pt}[90] % normal
\end{tikzpicture}

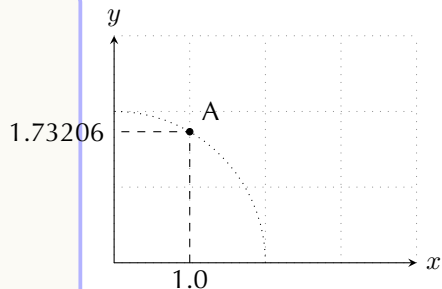
```



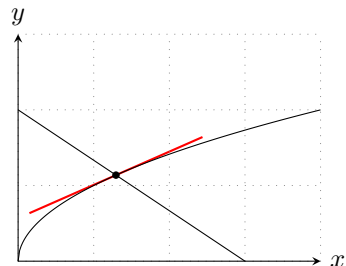


## 7.4 \tzgetxyval

```
% \tzgetxyval
\begin{tikzpicture}
\tzhelplines*(4,3)
\tzaxes(4,3){$x$}{$y$}
\tzcoord*(60:2cm)(A){A}[45]
\tzarc[dotted](0,0)(0:90:2cm)
\tzgetxyval(A){\Ax}{\Yx}
\tzproj[dashed](A){\Ax}{\Yx}
\end{tikzpicture}
```

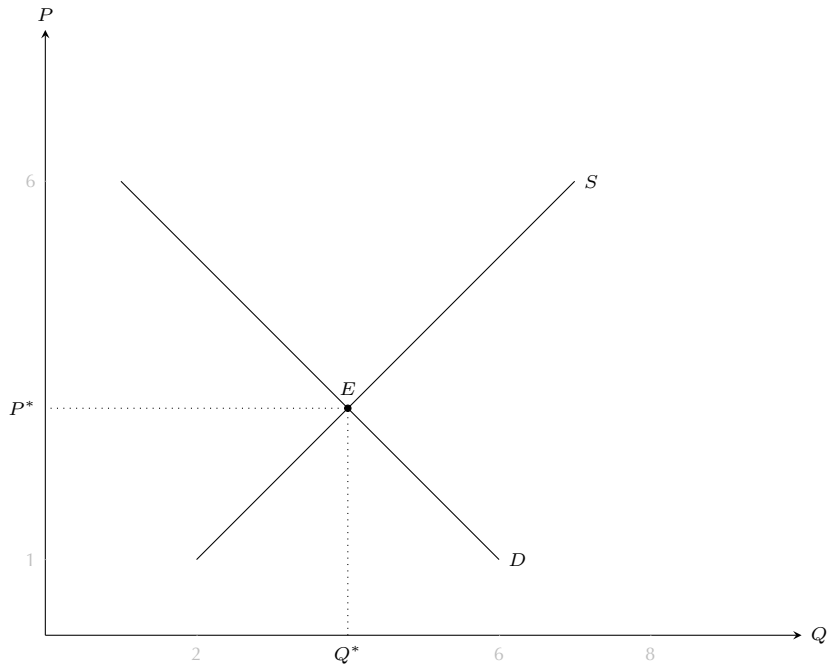


```
% \tzgetxyval
\begin{tikzpicture}
\tzhelplines*(4,3)
\tzaxes(4,3){$x$}{$y$}
\def\Fx{sqrt(\x)}
\tzfn\Fx[0:4]
\tzLFn"Gx"(0,2)(3,0)[0:3]
\tzXpoint*\Fx*\Gx*(B)
\tzgetxyval(B){\Bx}{\By}
\tzslope[red,thick]\Fx*(\Bx,0){2.5cm}
\end{tikzpicture}
```



## 8 Demand and Supply

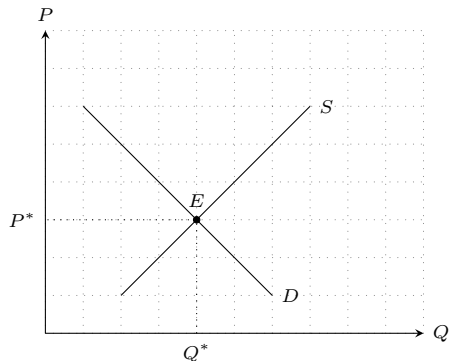
## 8.1 Demand and supply: Free drawing



```

% demand and supply: lines
\begin{tikzpicture}[scale=.5,font=\scriptsize]
% step 1
\tzhelplines(10,8)
\tzaxes(10,8){$Q$}{$P$}
% step 2
\tzto"dem"(1,6)(6,1){$D$}[r]
\tzto"supp"(2,1)(7,6){$S$}[r]
% step 3
\tzXpoint*{dem}{supp}(E){$E$}
\tzproj(E){$Q^*$}{$P^*$}
\end{tikzpicture}

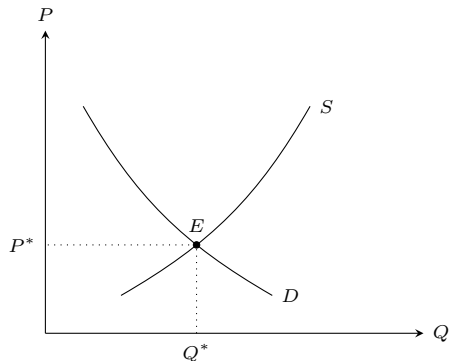
```



```

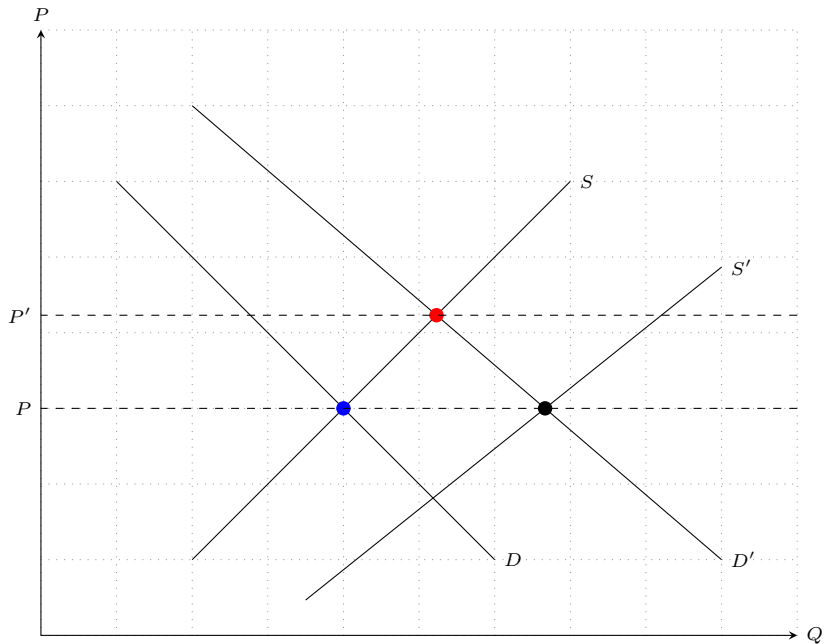
% demand and supply: curves
\begin{tikzpicture}[scale=.5,font=\scriptsize]
% step 1
%\tzhelplines(10,8)
\tzaxes(10,8){$Q$}{$P$}
% step 2
\tzto[bend right=15]"dem"(1,6)(6,1){$D$}[r]
\tzto[bend right=15]"supp"(2,1)(7,6){$S$}[r]
% step 3
\tzXpoint*{dem}{supp}(E){$E$}
\tzproj(E){$Q^*$}{$P^*$}
\end{tikzpicture}

```



## 8.2 Demand and supply: Changes

수요가 증가함에 따라 가격이 상승했다. 공급이 얼마만큼 증가해야 원래 가격을 돌아갈 수 있을까?

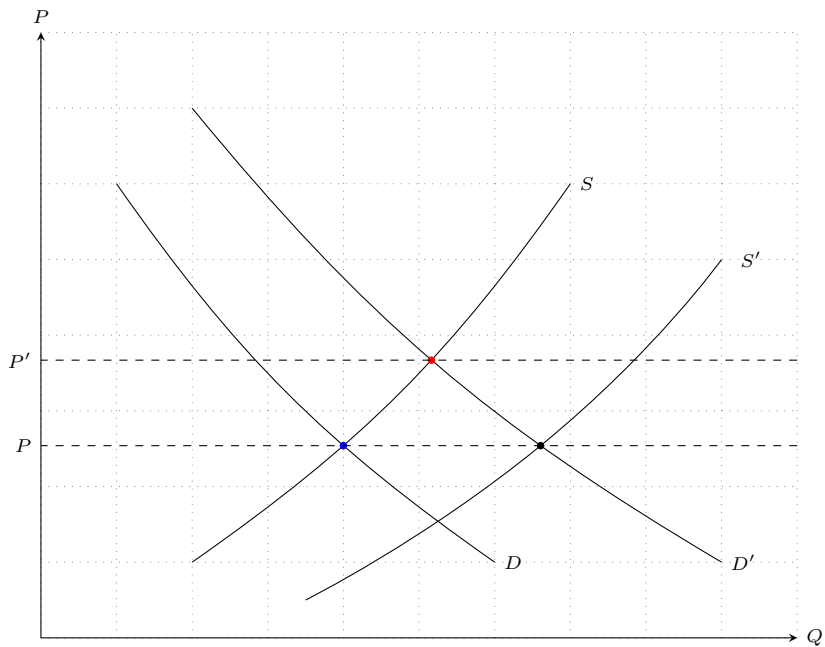


```

% demand and supply: changes: lines
\begin{tikzpicture}[scale=1,baseline,font=\scriptsize]
% step 1
\tzhelplines(10,8)
\setzdotsize{5pt}      %%
\tzaxes(10,8){$Q$}{$P$}
% step 2
\tzto"dem"(1,6)(6,1){$D$}[r]
\tzto"supp"(2,1)(7,6){$S$}[r]
\tzXpoint*[blue]{dem}{supp}(E)
% step 3
\tzto"demA"(2,7)(9,1){$D'$}[r]
\tzXpoint*[red]{demA}{supp}(E1)
% step 4
\tzhfn[dashed](E)[10:0]{$P$}[l]
\tzhfn[dashed](E1)[10:0]{$P'$}[l]
\tzhXpoint*{demA}(E)(EE)
% step 5
\tzLFn(EE){.8}[3.5:9]{$S'$}[r]
\end{tikzpicture}

```

수요곡선과 공급곡선을 곡선으로 그리고 싶다.





```

% curves
\begin{tikzpicture}[scale=1,baseline,font=\scriptsize,bend right=10]
% step 1
\tzhelplines(10,8)
%\settdotsize{5pt}      %%
\tzaxes(10,8){$Q$}{$P$}
% step 2
\tzto"dem"(1,6)(6,1){$D$}[r]
\tzto"supp"(2,1)(7,6){$S$}[r]
\tzXpoint*[blue]{dem}{supp}(E)
% step 3
\tzto"demA"(2,7)(9,1){$D'$}[r]
\tzXpoint*[red]{demA}{supp}(E1)
% step 4
\tzhfn[dashed](E)[10:0]{$P$}[l]
\tzhfn[dashed](E1)[10:0]{$P'$}[l]
\tzhXpoint*{demA}(E)(EE)
% step 5
\tzLFn(EE){.8}{3.5:9}{$S'$}[r]
\tzplotcurve(3.5,.5)(EE)(9,5){$S'$}[0];
\end{tikzpicture}

```

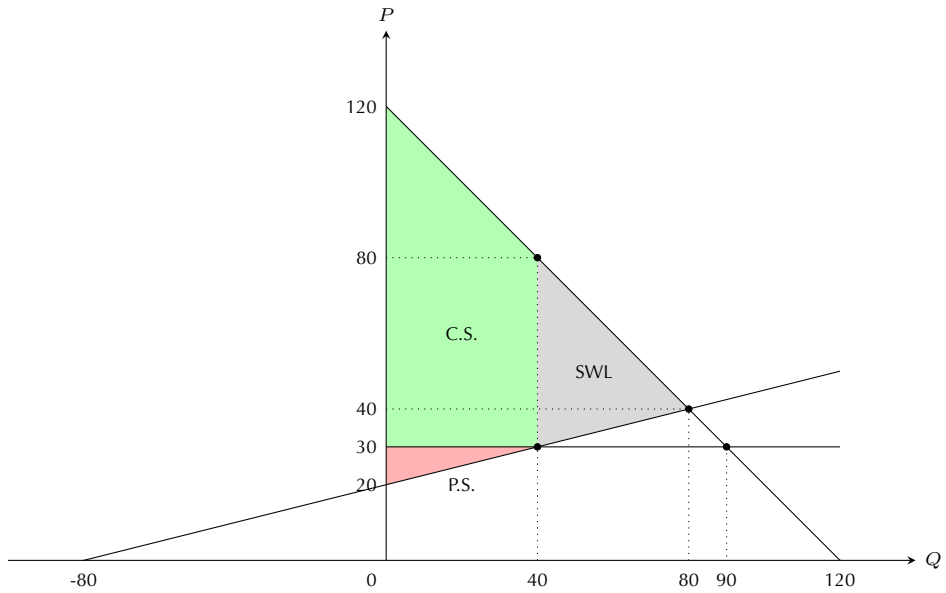
### 8.3 Consumer surplus and producer surplus

시장수요함수와 시장공급함수가 다음과 같은 완전경쟁시장을 고려하자.

$$D(P) = 120 - P \quad (\text{역수요함수 : } P(Q) = 120 - Q)$$

$$S(P) = 4P - 80 \quad (\text{역공급함수 : } P(Q) = 20 + \frac{1}{4}Q)$$

가격상한제가 실시되어  $\bar{P} = 30$ 으로 상한가격이 정해졌다고 할 때, 소비자 잉여(Consumer Surplus)와 생산자 잉여(Producer Surplus), 그리고 사회후생손실(Social Welfare Loss)을 그림으로 나타내라.



```

% \tzfnofy : function of \y
\begin{tikzpicture}[scale=.05,font=\scriptsize]
% step 1
%\tzhelp[step=10](-100,0)(140,140)
\tzaxes*(-100,0)(140,140){$Q$}{$P$}
\tzshoworigin
% step 2: define and plot functions
\def\Dy{120-\y}          \def\Sy{4*\y-80}          % 수요(공급)함수
\tzfnofy\Dy[0:120]      \tzfnofy\Sy[0:50]
\tzticks{-80,120}{20,120}
% step 3: intersection point
\tzXpoint{Dy}{Sy}(E)    \tzproj*(E){80}{40}
% step 4: price ceiling
\tzhfnat"maxP"{30}[0:120]{30}[at start,1]
\tzXpoint*{Sy}{maxP}(E1) \tzXpoint*{Dy}{maxP}(E2)
% step 5: vertical intersection point
\tzvXpoint*{Dy}(E1)(P)
\tzproj(P){40}{80}      \tzprojx(E2){90}
% step 6: CS, PS, SWL
\setztzpathlayer{behind}
\tzpath*[green](0,120)(P)(E1)(E1-|0,0);
\tzpath*[red](0,20)(E1)(E1-|0,0);
\tzpath*(E)(P)(E1);
\tznode(20,60){C.S.}    \tznode(20,20){P.S.}    \tznode(55,50){SWL}
\end{tikzpicture}

```

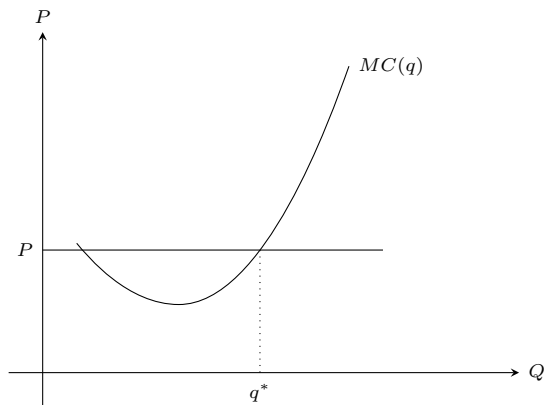
```

% \tzfn : function of \x
\begin{tikzpicture}[scale=.05,font=\scriptsize]
% step 1
\tzhelplines[step=10](-100,0)(140,140)
\tzaxes*(-100,0)(140,140){$Q$}{$P$}
\tzshoworigin
% step 2: define and plot functions
\def\Dx{120-\x}          \def\Sx{20+1/4*\x}      % 역수요(공급)함수
\tzfn\Dx[0:120]          \tzfn\Sx[-80:120]
\tzticks{-80,120}{20,120}
% step 3: intersection point
\tzXpoint{Dx}{Sx}(E)    \tzproj*(E){80}{40}
% step 4: price ceiling
\tzhfnat"maxP"{30}[0:120]{30}[at start,1]
\tzXpoint*{Sx}{maxP}(E1) \tzXpoint*{Dx}{maxP}(E2)
% step 5: vertical intersection point
\tzvXpoint*{Dx}(E1)(P)
\tzproj(P){40}{80}      \tzprojx(E2){90}
% step 6: CS, PS, SWL
\setztzpathlayer{behind}
\tzpath*[green](0,120)(P)(E1)(E1-|0,0);
\tzpath*[red](0,20)(E1)(E1-|0,0);
\tzpath*(E)(P)(E1);
\tznode(20,60){CS}      \tznode(20,20){PS}      \tznode(55,50){SWL}
\end{tikzpicture}

```

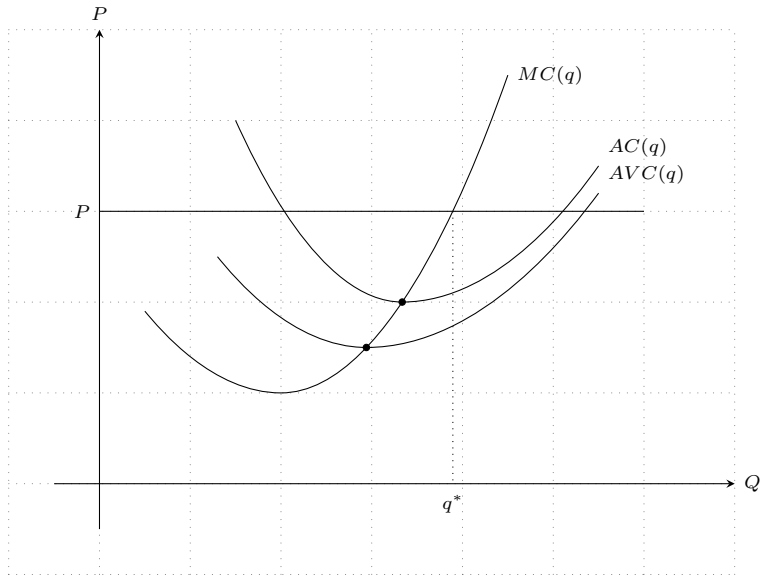
## 9 Profit maximization

## 9.1 Profit maximization: Competitive firm



```
% profit maximization: P=MC
\begin{tikzpicture}[scale=.9,font=\scriptsize]
\i{tzhelplines(-1,-1)(7,5)}
\tzaxes(-.5,-.5)(7,5){$Q$}{$P$}
\tzparabola"MC"(.5,1.9)(2,1)(4.5,4.5){$MC(q)$}[r]
\tzhfnat"price"{1.8}[0:5){$P$}[at start,1]
\tzXpoint{price}{MC}(E)
\tzprojx(E-2){$q^*$} % use the second intersection
\end{tikzpicture}
```

## 9.2 Cost curves



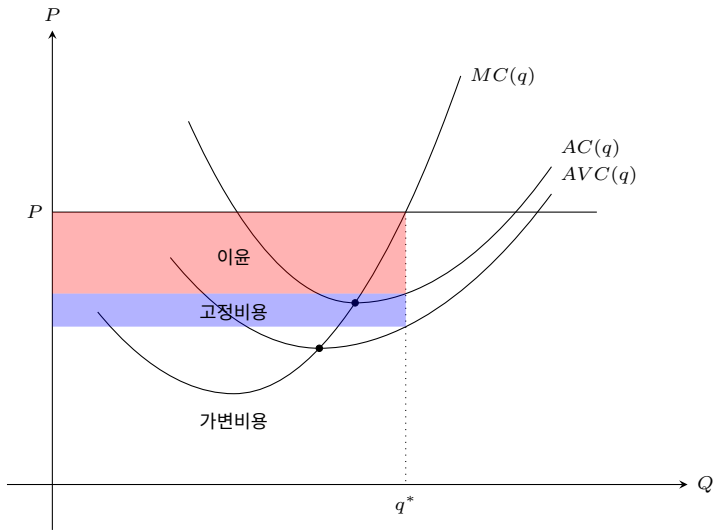


```

% cost curves: MC, AC, AVC curves
\begin{tikzpicture}[scale=1.2,font=\scriptsize]
\tzhelpelines(-1,-1)(7,5)
\tzaxes(-.5,-.5)(7,5){$Q$}{$P$}
\tzparabola"MC"(.5,1.9)(2,1)(4.5,4.5){$MC(q)$}[r]
\tzhfmat"price"{3}[0:6){$P$}[at start,1]
\tzXpoint{price}{MC}(E)
\tzprojx(E){$q^*$}
%
\tzXpointat*{MC}{2}(minAC)
\tzparabola(1.5,4)(minAC)(5.5,3.5){$AC(q)$}[ar]
\tzXpointat*{MC}{1.5}(minAVC)[2] % second intersection
\tzparabola(1.3,2.5)(minAVC)(5.5,3.2){$AVC(q)$}[ar]

\end{tikzpicture}

```

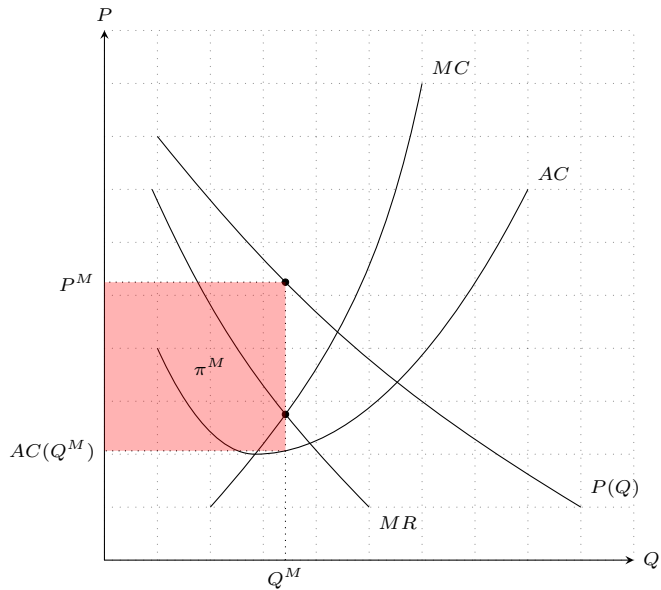


```

% cost curves: profit
\begin{tikzpicture}[scale=1.2,font=\scriptsize]
\tzhelplines[draw=none](-1,-1)(7,5)
\tzaxes(-.5,-.5)(7,5){$Q$}{$P$}
\tzparabola"MC"(.5,1.9)(2,1)(4.5,4.5){$MC(q)$}[r]
\tzhfmat"price"{3}[0:6){$P$}[at start,1]
\tzXpoint{price}{MC}(E)
\tzprojx(E){$q^*$}
%
\tzXpointat*{MC}{2}(minAC)
\tzparabola"AC"(1.5,4)(minAC)(5.5,3.5){$AC(q)$}[ar]
\tzXpointat*{MC}{1.5}(minAVC)[2] % second intersection
\tzparabola"AVC"(1.3,2.5)(minAVC)(5.5,3.2){$AVC(q)$}[ar]
%
\tzvXpoint{AC}(E)(ACq)
\tzvXpoint{AVC}(E)(AVCq)
\tzpath*[red](E)(ACq)(ACq-|0,0)(E-|0,0);
\tzpath*[blue](ACq)(AVCq)(AVCq-|0,0)(ACq-|0,0);
\tznode(2,2.5){이윤}
\tznode(2,1.9){고정비용}
\tznode(2,0.7){가변비용}
\end{tikzpicture}

```

### 9.3 Profit maximization: Monopoly



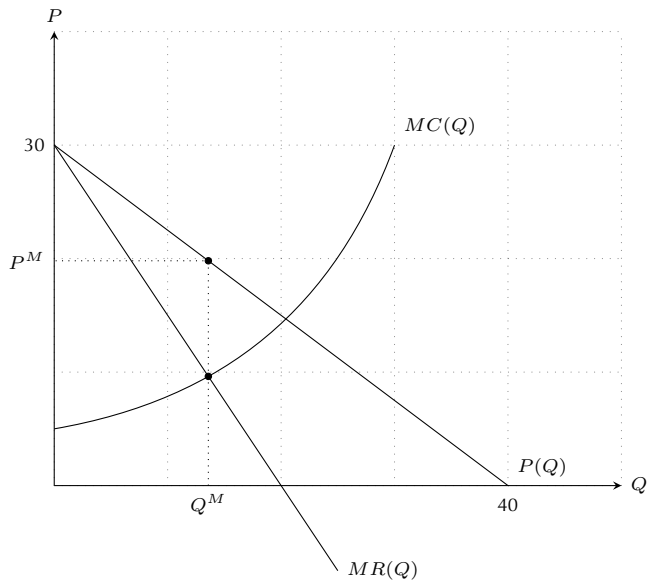
```

% monopoly: equilibrium
\begin{tikzpicture}[scale=.7,font=\scriptsize]
% step 1: determine the picture size
\tzhelplines(10,10)
\tzaxes(0,0)(10,10){$Q$}{$P$}
% step 2: draw graphs
\tzto[bend right=10]"dem"(1,8)(9,1){$P(Q)$}[ar]
\tzto[bend right=10]"MR"(.9,7)(5,1){$MR$}[br]
\tzto[bend right=15]"MC"(2,1)(6,9){$MC$}[ar]
\tzhXpointat{MC}{2}(minAC)
\tzparabola"AC"(1,4)(minAC)(8,7){$AC$}[ar]
% step 3: equilibrium
\tzXpoint*{MR}{MC}(ME)
\tzvXpoint*{dem}(ME)(MP)
\tzproj(MP){$Q^M$}{$P^M$}
% step 4: equilibrium profit
\tzvXpoint{AC}(ME)(ACq)
\tzprojy(ACq){$AC(Q^M)$}
\tzpath*[red](MP)(MP-|0,0)(ACq-|0,0)(ACq);
\tznode(2,4){$\pi^M$}[b]
\end{tikzpicture}

```

독점시장의 역수요함수가  $P(Q) = 30 - \frac{3}{4}Q$ 이라 하자. 그러면 한계수입은  $MR(Q) = 30 - \frac{3}{2}Q$ 이다.

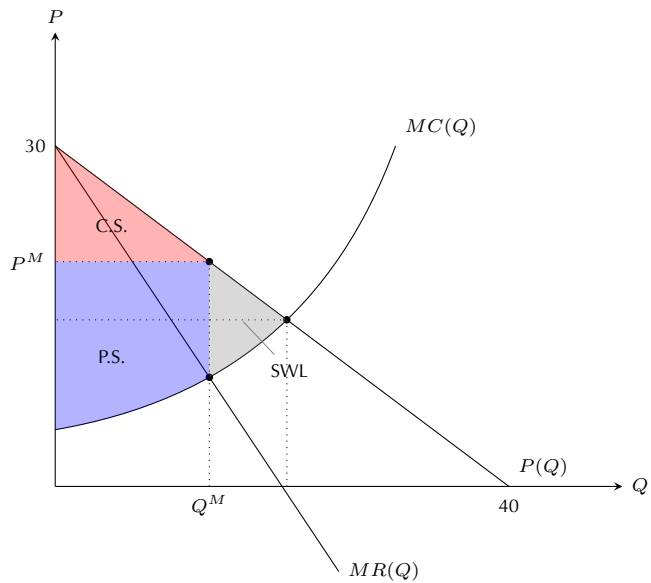
독점시장 균형을 구하고, 소비자 잉여(C.S.), 생산자 잉여(P.S.), 사회후생손실(SWL)를 나타내라.



```

% monopoly: welfare analysis
\begin{tikzpicture}[scale=.15,font=\scriptsize]
% step 1: determine the picture size
\tzhelplines[step=10](50,40)
\tzaxes(0,0)(50,40){$Q$}{$P$}
% step 2: draw graphs
\def\dem{30-3/4*\x}
\def\MR{30-3/2*\x}
\tzfn\dem[0:40]{$P(Q)$}[ar]
\tzfn\MR[0:25]{$MR(Q)$}[r]
\tzto[bend right]"MC"(0,5)(30,30){$MC(Q)$}[ar]
\zticks{40}{30}
% step 3: monopoly equilibrium
\tzXpoint*\MR*\MC(ME)
\tzvXpoint\dem(ME)(MP)
\tzproj*(MP){$Q^M$}{$P^M$}
\end{tikzpicture}

```





```

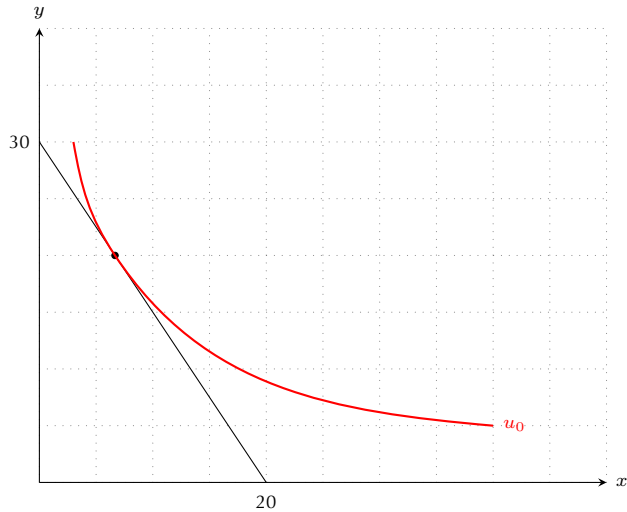
% monopoly: welfare analysis
\begin{tikzpicture}[scale=.15,font=\scriptsize]
% step 1: determine the picture size
%\tzhelplines[step=10](50,40)
\tzaxes(0,0)(50,40){$Q$}{$P$}
% step 2: draw graphs
\def\dem{30-3/4*\x}          \def\MR{30-3/2*\x}
\tzfn\dem[0:40]{$P(Q)$}[ar]  \tzfn\MR[0:25]{$MR(Q)$}[r]
\tzto[bend right]"MC"(0,5)(30,30){$MC(Q)$}[ar]
\tzticks{40}{30}
% step 3: monopoly equilibrium
\tzXpoint*\MR*\MC*(ME)
\tzvXpoint\dem*(ME)(MP)
\tzproj*(MP){$Q^M$}{$P^M$}
% step 4: competitive equilibrium
\tzXpoint\dem*\MC*(CE)
\tzproj*(CE)
% step 5: filling
\tzpath*[red](0,30)(MP)(MP-|0,0);
\tzpath*[blue](0,5)[to[bend right=9]](ME)(MP)(MP-|0,0);
\tzpath*(ME)[to[bend right=7]](CE)(MP);
% step 6: labelling
\tznode(5,23){C.S.}
\tznode(5,10){P.S.}[a]
\tznode($ (CE)!4cm!(CE-|0,0)$){}[inner sep=0pt,pin={-60:SWL}]
\end{tikzpicture}

```

## 10 **Utility maximization**

## 10.1 Consumer equilibrium

예산제약:  $3x + 2y = 60$ 일 때, 효용극대점을 그림으로 나타내라. (즉, 주어진 직선에 접하는 곡선을 그려라.)



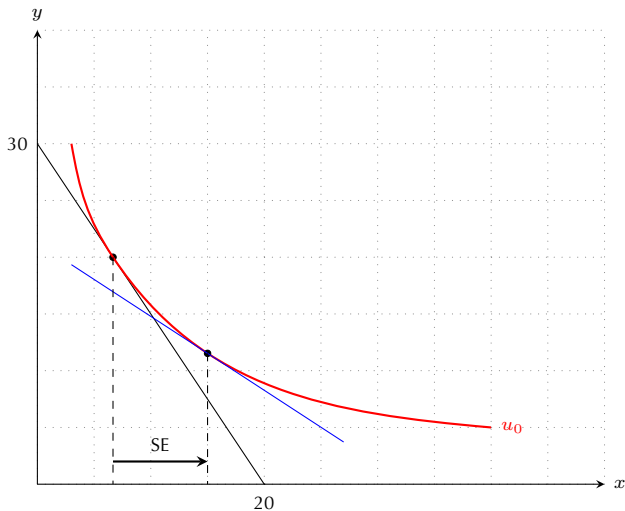
```

\begin{tikzpicture}[scale=.15,font=\scriptsize]
\tzhelplines[step=5](50,40)
\tzaxes(50,40){$x$}{$y$}
\zticks{20}{30}
\def\bgt{30-3/2*\x}
\tzfn\bgt[0:20]
\tzXpointat*\bgt}{20}(E)
\tzcoors(20,0)(X)(0,30)(Y);
\tzpointangle(X)(Y){\xx}
\ztos[thick,red](3,30)[out=-80,in=\xx](E)[out=\xx-180,in=175](40,5){$u_0$}[r];
\end{tikzpicture}

```

## 10.2 Substitution effect 1

이제,  $x$  재의 가격이 하락했다고 하자. 대체효과의 크기를 그림으로 나타내라. (즉, 주어진 곡선에 접하는 직선을 그려라.)

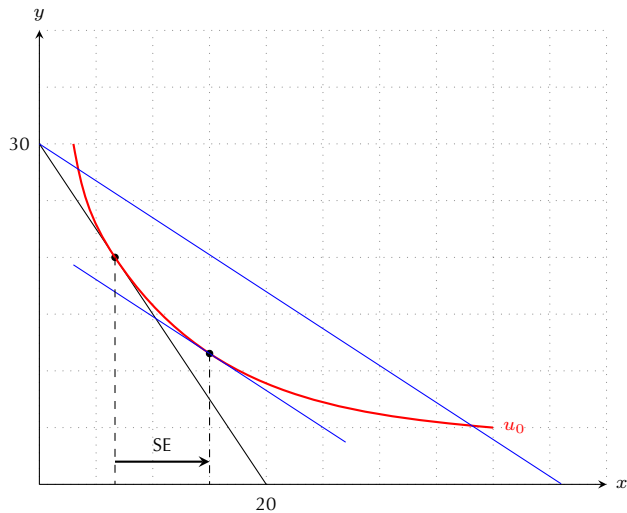


```

\begin{tikzpicture}[scale=.15,font=\scriptsize]
\tzhelpines[step=5](50,40)
\tzaxes(50,40){$x$}{$y$}
\ztzticks{20}{30}
% budget line
\def\bgt{30-3/2*\x}
\tzfn\bgt[0:20]
% equilibrium point
\tzHXpointat*\bgt}{20}(E)
% IC curve
\tzcoors(20,0)(X)(0,30)(Y);
\tzpointangle(X)(Y){\xx}
\tztos[thick,red]"IC"(3,30)[out=-80,in=\xx](E)[out=\xx-180,in=175](40,5){$u_0$}[r];
% tangent line
\setztangentlayer{main}
\tzvXpointat*\{IC\}{15}(K)
\ztztangentat[blue]{IC}{15}(.3,.2)[3:27] % (epsilon_1,epsilon_2)
% substitution effect
\tzprojx[dashed](E)
\tzprojx[dashed](K)
\tzline[->,thick]<0,2>(E|-0,0){SE}(K|-0,0)
\end{tikzpicture}

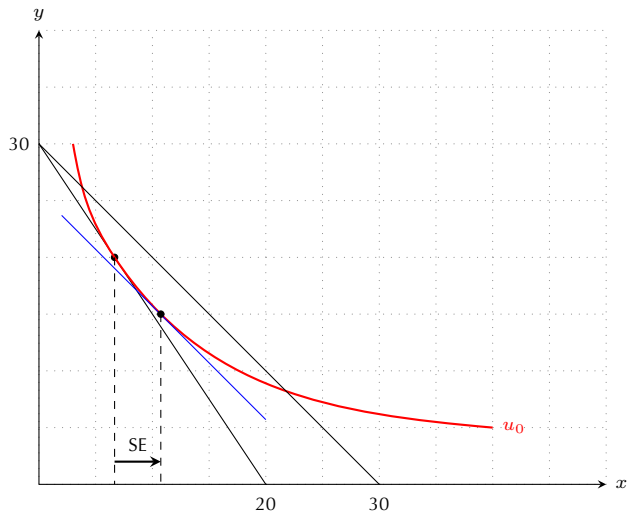
```

Question: 새로운 예산선(blue line)은 어떻게 그렸을까?



### 10.3 Substitution effect 2

이제,  $x$ 재의 가격이  $p_x = 2$ 로 하락했다고 하자. 대체효과의 크기를 그림으로 나타내라. (즉, 주어진 곡선에 주어진 기울기를 갖는 접선을 그려라.)





```

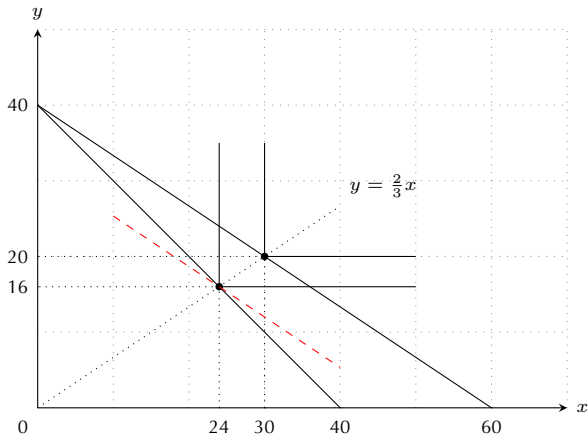
\begin{tikzpicture}[scale=.15,font=\scriptsize]
\tzhelplices[step=5](50,40)
\tzaxes(50,40){$x$}{$y$}
\ztzticks{20,30}{30}
% budget line
\def\bgt{30-3/2*\x}
\tzfn\bgt[0:20]
% equilibrium point
\tzhXpointat*\bgt{20}(E)
% IC curve
\tzcoors(20,0)(X)(0,30)(Y);
\tzpointangle(X)(Y){\xx}
\ztzos[thick,red]"IC"(3,30)[out=-80,in=\xx](E)[out=\xx-180,in=175](40,5){$u_0$}[r];
% new budget line
\def\bgtA{30-\x}
\tzfn\bgtA[0:30]
% shifted tangent line
\tzfn[blue]<0,-4.3>\bgtA[2:20] % trial and error
\tzhXpointat*\bgtA{15}(K)
% substitution effect
\tzprojx[dashed](E)
\tzprojx[dashed](K)
\tzline[->,thick]<0,2>(E|-0,0){SE}(K|-0,0)
\end{tikzpicture}

```

## 10.4 Utility maximization: Leontief utility

Comparative statics:

- 효용함수:  $u(x, y) = \min\{2x, 3y\}$
- 예산제약:  $3x + 3y = 120$
- 가격변화:  $x$  재의 가격이  $p_x = 2$ 에서  $p'_x = 3$ 으로 하락



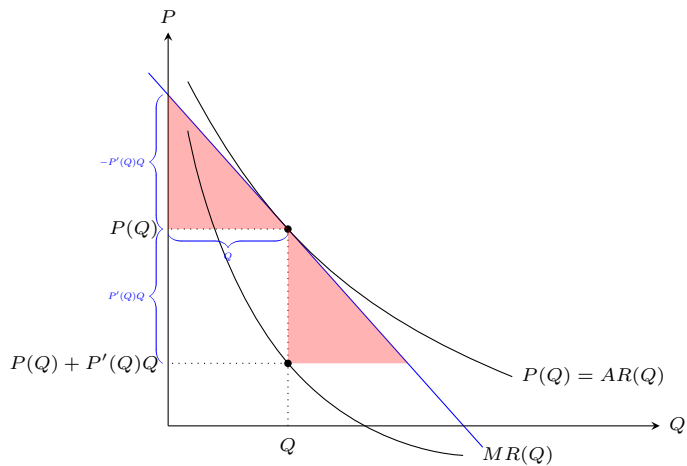
```

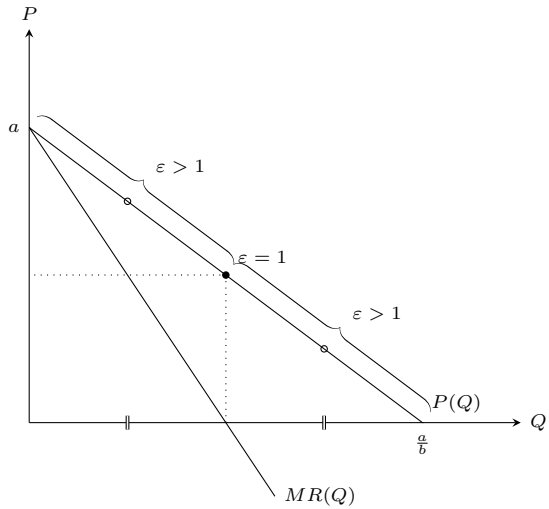
% Leontief utility
\begin{tikzpicture}[scale=.1,font=\scriptsize]
\tzhelplices[step=10](70,50)
\tzaxes(0,0)(70,50){$x$}{$y$}
\tzshoworigin
\zticks{24,30,40,60}{16,20,40}
% ray
\def\Fx{2*\x/3}
\tzfn[dotted]{\Fx}[0:40]{$y=\frac{2}{3} x$}[ar]
% original equilibrium
\def\bgt{40-\x}
\tzfn{\bgt}[0:40]
\tzaxesL(24,16)(50,35)
\tzXpoint*{\bgt}{\Fx}(A)
\tzproj(A)
% new equilibrium
\def\bgtb{40-2/3*\x}
\tzfn{\bgtb}[0:60]
\tzaxesL(30,20)(50,35)
\tzXpoint*{\bgtb}{\Fx}(B)
\tzproj(B)
% price effect
\tzLFn[red,dashed](24,16){-40/60}[10:40]
\end{tikzpicture}

```

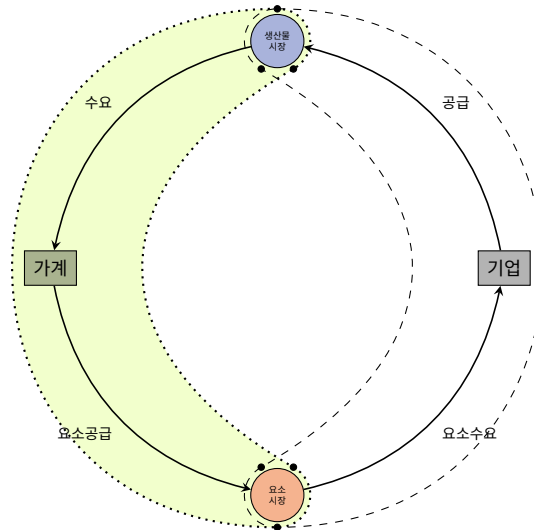
## 11 **More examples**

## 11.1 Marginal Revenue and Elasticity

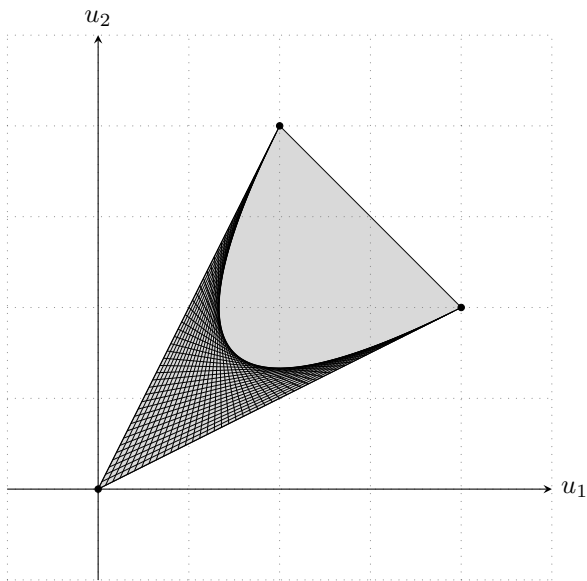




## 11.2 Circular flow model

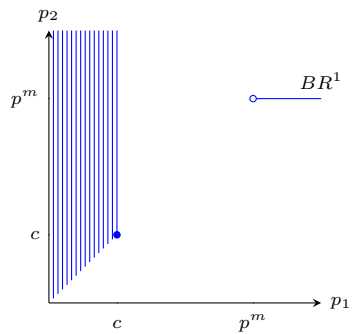
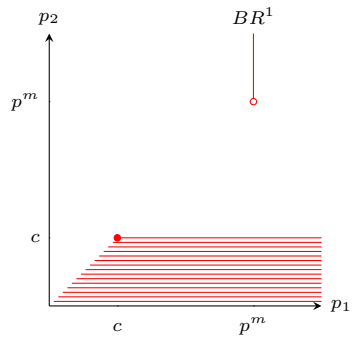


### 11.3 Mixed strategy Nash equilibrium payoffs: foreach

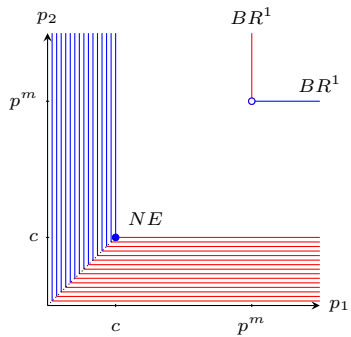




## 11.4 Cournot Duopoly: Best response analysis



Thus, NE is:  $(p_1^*, p_2^*) = (c, c)$ .



## 12 References

- The TikZ and PGF Package: Manual for version 3.1.9a  
(<https://github.com/pgf-tikz/pgf>).
- `tzplot.sty`: Plot Graphs with TikZ Abbreviations, version 2.0  
(<https://www.ctan.org/pkg/tzplot>).
- 경제학자를 위한 TikZ  
(<http://wiki.ktug.org/wiki/wiki.php/LaTeXWorkshop/2017>).
- `tzplot`: Basics  
(<http://wiki.ktug.org/wiki/wiki.php/LaTeXWorkshop/2021>)
- `tzplot`: How to Plot Graphs  
(<http://wiki.ktug.org/wiki/wiki.php/LaTeXWorkshop/2021>)